The measured amplitudes of the reflected and transmitted signals are proportional to the amplitude of the matrix elements in the first column of the matrix. Linear combination of the incident and reflected signals in a bridge circuit makes it possible to generate a signal with similar form for which the values of the constants are adjustable. If the bridge is balanced off resonance, that is with $z=0$, the constant in the numerator drops out.

The term $(1+s_{33})$ in the denominator represents the observed line broadening associated with the radiation of energy from the sample. If the $(1+s_{33})$ term is not negligible, the observations of linewidth are made difficult by the influence of the coupling of the sample to the waveguide. It is possible for the coupling to be large enough to make precision linewidth measurements a practical impossibility. One method of eliminating this difficulty is to use the transmitted signal as the reference throughout the experiment. This can be accomplished by appropriately adjusting the incident amplitude. The reflected signal R is the ratio of the terms in the first column of the matrix (1).

$$R = \frac{(s_{11} - s_{22}^*) + (s_{11} + s_{22}^*)z}{(s_{21} + s_{12}^*) + (s_{21} - s_{12}^*)z}. \quad (2)$$

The constant, $(s_{11}-s_{22}^*)$, in the numerator drops out when the bridge is balanced far off resonance since R must then be zero. The denominator term $(s_{21}-s_{12}^*)$ is of special interest since this term vanishes if two restrictions are applied to the original S matrix. The first is that the matrix be symmetric. This is easily fulfilled and tested physically since this is identical to the requirement of reciprocal coupling. It is possible to test this in a magnetic resonance experiment by reversing the magnetic field and observing the reciprocity of the resonant properties.

The other condition requires that the three-port matrix S be one in which, by adjusting only the phase reference at the external ports, it is possible simultaneously to obtain real, positive values for both the determinant of the S matrix and the matrix component s_{21}. In a physical analysis, this is equivalent to the removal of wall effects, or to the condition that there is no shift of the resonant frequency due to coupling of the waveguide system. A frequency shift can be associated with a reaction loading by the transmission line. Only if the *reactive* loading is kept small will we be able to compensate for the *resistive* loading effect by the waveguide system.

$$R = \frac{s_{11} + s_{22}^*}{s_{21} + s_{12}^*} z. \quad (3)$$

The final form of the reflected signal (3) is obtained by applying these requirements; the system is reciprocally coupled, the bridge is balanced off resonance so that $z=0$, the transmitted signal is held constant, and the sample is not located near the waveguide wall. Since R is proportional to z, the observed linewidth of R is likewise the intrinsic linewidth of the sample.

The dependence of the apparent linewidth upon an error in satisfying the reciprocity condition was tested experimentally. The linewidth of the reflected signal *did* vary somewhat with change in position of the sample in the waveguide; however, there was no *pronounced* sensitivity in the observed linewidth. The precision to which the reciprocity condition could be satisfied was sufficient to warrant use of this system. The final system error was much greater than the residual error associated with position error.

The microwave bridge circuit used to observe the resonance linewidth is shown in Fig. 2. The klystron stabilization held the signal frequency to a preset value with only a small residual frequency modulation. The reference and reflection signal branches were coupled to the main branch through 10-db and 3-db directional couplers, respectively, and were mixed for bridge operation in the 3-db coupler. The modulation servo-amplifier allowed for automatic compensation for transmission line loading upon the resonant sample. The test section shown in Fig. 3 was constructed of RG 52/U waveguide.

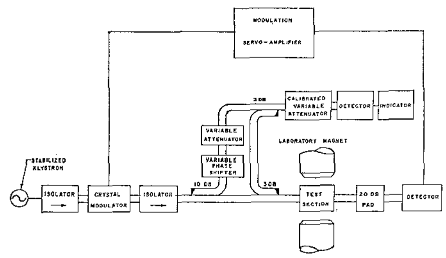

Fig. 2—Block diagram of the measurement system.

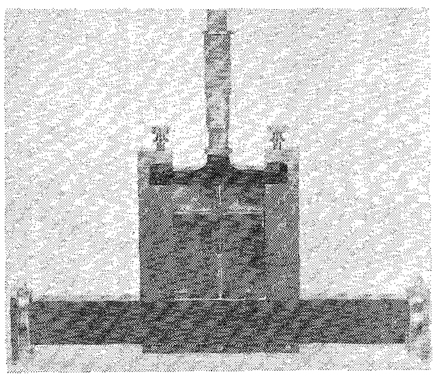

Fig. 3—Photograph of the test section with sample holder in place.

The reflection bridge was balanced by adjustment of the phase shift and attenuation on the reference arm. Measurements were made by observing the magnetic field difference corresponding to the 3-db response width of the reflection bridge signal. The magnetic field was measured with suitable precision using a nuclear magnetic resonance gaussmeter and a frequency counter for narrow linewidths. For narrow linewidths the final precision was ±0.02 oersted.

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A Note on Strip-Line Band-Stop Filters with Narrow Stop Bands*

Design criteria for band-stop filters having bandwidths up to few per cent have been presented by Young, Matthaei, and Jones. The purpose of this correspondence is to extend their work to include a new type of strip-line resonator which is easier to construct and adjust, and also permits the application of printed circuit fabrication.

Fig. 1 shows the basic structure of the band-stop filter to be considered corresponding to Fig. (3a) of the paper by Young, *et al.*[1] Fig. 2 shows a schematic of how the circuit of Fig. 1 may be realized in practice corresponding to Fig. 5 of the cited reference. However, resonant circuits of the type shown require considerable care in adjustment as noted by the design example given by Young, *et al.* The major difficulties are in determining the gap spacing required to obtain the proper value of resonator capacitance and in establishing the reference planes for the stub lengths. The type of resonator shown in Fig. 3 and which is the subject of

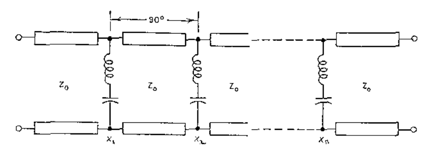

Fig. 1—Quarter wave coupled shunt resonator band-stop filter.

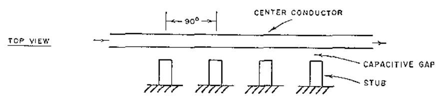

Fig. 2—Physical center conductor geometry for strip-line filter.

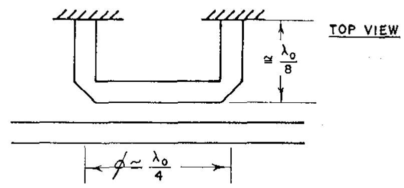

Fig. 3—Parallel coupled line resonator.

this note will eliminate these difficulties. The only adjustment required is the position of the short circuit which is made necessary to compensate for the mitered corners. Only the case where all transmission lines have equal characteristic impedances is considered. This limits the design criteria to applications where a maximum flat response or Tchebyscheff response with an odd number of resonators is desired. The extension to the general case is readily accomplished, but will not be considered here.

* Received July 17, 1963. Based on part of the research work undertaken by Robert Dean Standley in partial fulfillment of the requirements for the Ph.D. degree at Illinois Institute of Technology, Chicago, Ill.

[1] L. Young, *et al.*, "Microwave band-stop filters with narrow stop bands," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-10, pp. 416–428; November, 1962.

The procedure used to establish design criteria for band-stop filters utilizing the resonators of Fig. 3 is straightforward. The basic section of the filter of Fig. 1 is considered to be a quarter-wavelength transmission line shunted at its mid-point by a series resonant circuit as shown in Fig. 4. The image parameters of the resonators of Figs. 3 and 4 are then equated. The slope parameter of the shunt series-resonant circuit is then related to the coupling parameter of the parallel coupled lines.

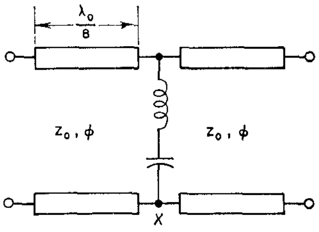

Fig. 4—Basic section of filter in Fig. 1.

For the resonator of Fig. 3 the image parameters are

$$Z_I = Z_0\sqrt{\frac{\tan^2\phi - k}{k\tan^2\phi - 1}}, \qquad (1)$$

$$\gamma_I = 2\tanh^{-1} \cdot \left\{\tan\phi\left[\frac{1-k\tan^2\phi}{\tan^2\phi - k}\right]^{1/2}\right\}, \qquad (2)$$

where

$$k = \sqrt{1-c^2} \simeq 1 - c^2/2 \quad \text{for } c^2 \ll 1, \qquad (3)$$

$$c = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} = \text{coupling parameter}, \qquad (4)$$

Z_{oe} = even-mode characteristic impedance with respect to ground of each conductor.

Z_{oo} = odd-mode characteristic impedance with respect to ground of each conductor.

ϕ is defined in Fig. 3.

For the resonator of Fig. 4 the image parameters are

$$Z_I' = Z_0\sqrt{\frac{\tan\phi(\tan\phi + y)}{y\tan\phi - 1}}, \qquad (5)$$

$$\gamma_I' = 2\tanh^{-1}\tan\phi \cdot \left[\frac{1 - y\tan\phi}{\tan\phi(\tan\phi + y)}\right]^{1/2}, \qquad (6)$$

where

$$y = \frac{2}{Z_0}\left(\omega L - \frac{1}{\omega C}\right). \qquad (7)$$

Thus if $Z_I = Z_I'$, then $\gamma_I = \gamma_I'$. Equating the image impedances yields

$$y = \frac{2 - c^2}{c^2}(1 - \cot^2\phi). \qquad (8)$$

The slope parameter of the shunt series-resonant circuit is given by

$$x = \frac{\omega_0}{2}\left.\frac{dX}{d\omega}\right|_{\omega=\omega_0} \qquad (9)$$

and is given in terms of the low-pass prototype elements by Young, *et al.* Using (7) and (8)

$$\left.\frac{dy}{d\omega}\right|_{\omega=\omega_0} = \frac{4}{\omega_0}\frac{x}{Z_0} \qquad (10)$$

$$= \frac{2-c^2}{c^2}\frac{\pi}{\omega_0}. \qquad (11)$$

Therefore,

$$\frac{c^2}{2-c^2} = \frac{\pi}{4}\left(\frac{Z_0}{x}\right)$$

or

$$c^2 = \frac{\frac{\pi}{2}\left(\frac{Z_0}{x}\right)}{1 + \frac{\pi}{4}\frac{Z_0}{x}}. \qquad (12)$$

Using the expressions for x_i, the slope parameter for the ith shunt resonator, given by Young,

$$c_1 = \sqrt{\frac{2\pi\omega_1' g_o g_1}{4w^{-1} + \pi\omega_1' g_o g_1}}; \qquad (13)$$

$$c_i = \sqrt{\frac{2\pi\omega_1' g_i}{4g_o w^{-1} + \pi\omega_1' g_i}} \quad \text{for } i\text{-even}, \qquad (14)$$

$$c_i = \sqrt{\frac{2\pi\omega_1' g_o g_i}{4w^{-1} + \pi\omega_1' g_i g_o}} \quad \text{for } i\text{-odd}. \qquad (15)$$

This completes the derivation of the design criteria. Identical results are obtained by considering the behavior of the resonators near resonance, using the expressions for L and C given by Young, *et al.*, in terms of the low pass prototype, and satisfying (8). In this case the approximations required are

$$\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \simeq 2\left(\frac{\omega - \omega_0}{\omega_0}\right) \qquad (16)$$

and

$$\left[1 + \frac{\pi}{4}\left(\frac{\omega - \omega_0}{\omega_0}\right)\right]^2 \simeq 1 + \frac{\pi}{2}\left(\frac{\omega - \omega_0}{\omega_0}\right). \qquad (17)$$

These approximations are quite valid for narrow bandwidth filters.

It should also be noted that a second type of resonator similar to that shown in Fig. 3 may also be used. For the latter the short circuits in Fig. 3 are replaced by open circuits. The design criteria may be developed in a manner similar to the above. However, this type of resonator requires compensation due to fringing at the open circuit ends. In addition some sort of support such as a dielectric post is required which tends to decrease unloaded Q. Hence, it is believed that the resonator of Fig. 3 is more suitable for practical applications.

Model work on filters of this type will begin shortly.

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Comments on "Maximum Efficiency of a Two Arm Waveguide Junction"*

In connection with a recent communication by Beatty[1] I wish to advise you that we have been studying the two-arm dissipative waveguide junction in our laboratory. Two years ago I published[2,3] a new demonstration of Deschamp's method for measuring scattering coefficients and the general properties of those coefficients.

In a recent work, not yet published, we show the variations of the absorbed power against the reflection coefficient of the terminal load.

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* Received July 22, 1963.

[1] R. W. Beatty, "Maximum efficiency of a two arm waveguide junction," IEEE Trans. on Microwave Theory and Techniques (*Correspondence*), vol. MTT-11, p. 94; January, 1963.

[2] S. Lefeuvre, "Détermination de l'impédance caractéristique d'un quadripole quelconque en hyperfréquences," *Compt. Rend. Acad. Sci.*, vol. 250, pp. 3288–3289, 1960.

[3] S. Lefeuvre, "Quelque propriétés des quadripoles dissipatifs en hyperfréquences," *Compt. Rend. Acad. Sci.*, vol. 252, pp. 4135–4136; 1961.

A Uniform Coaxial Line with an Elliptic-Circular Cross Section*

Analysis and design of a nonuniform coaxial line with an isoperimetric sheath deformation has been reported.[1] The object of this note is to show that the procedure followed therein can be adopted for evaluating some of the essential features of an infinitely long ideal transmission line with an elliptic sheath and a circular inner conductor. Apart from its reported use with the nonuniform line, this type of structure may also be found in medium and large sized electronuclear machines.[2]

I. Line Constants

Eqs. (15)–(21) in the communication quoted[1] provide expressions for the primary and the secondary constants of the line as follows:

$$\text{Capacitance per unit length} = C = \frac{4\pi\epsilon}{G} \qquad (1a)$$

External inductance per unit length

$$= l^e = \frac{\mu G}{4\pi} \qquad (1b)$$

Characteristic impedance

$$= Z_0 = \frac{G}{4\pi}\sqrt{\frac{\mu}{\epsilon}} \qquad (2a)$$

* Received March 11, 1963; revised manuscript received July 23, 1963.

[1] N. Seshagiri, "A non-uniform line with an isoperimetric sheath deformation," this issue, page 478.

[2] P. M. Morse and H. Feshbach, "Methods of Theoretical Physics," McGraw-Hill Book Co., Inc., New York, N. Y., p. 1204; 1953.